Pattern Matching under Weighted Edit Distance

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United Kingdom

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Saarbrücken, Germany

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National Institute of Informatics
Tokyo, Japan

FOCS 2025

An Example

...

(TU Kaiserslautern).

(Simons Institute and UC Berkeley), (University of Salzburg), (University of Salzburg).

(University of Pennsylvania), (University of Pennsylvania), (University of Pennsylvania).

(Reichman University, Herzliya, Israel and Birkbeck, University of London), (UC Berkeley and Max Planck Institute for Informatics, SIC, Saarbrücken, Germany), (Max Planck Institute for Informatics, SIC, Saarbrücken, Germany).

(National Institute of Informatics).

(UniversitÃ© Paris CitÃ©, CNRS, IRIF, F-75013, Paris, France); (Universitat PolitÃ¨cnica de Catalunya)

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(ETH Zurich); (Toyota Technological Institute at Chicago)

(Merton College, University of Oxford, United Kingdom); (Mathematical Institute, University of Bonn, Germany); (Max Planck Institute for Informatics, Saarland Informatics Campus (SIC), Saarbr|cken, Germany)

(The Academic College of Tel Aviv-Yafo), (UC Berkeley), (Weizmann Institute of Science), (University of California San Diego).

(CISPA Helmholtz Center for Information Security, Saarbrücken); (Simon Fraser University); (Duke University); (CISPA Helmholtz Center for Information Security, Saarbrücken); (Max Planck Institute for Informatics, SIC, Saarbrücken)

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Task: Find **Saarbrücken** in a text.

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Or Saarbruecken.

Task: Find **Saarbrücken** in a text.

Or Saarbruecken. Or Sarrebruck.

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Or Saarbruecken. Or Sarrebruck. Or Saarbrucken, Saarbr|cken, SaarbrÃ¼cken, ...

early 1980's

Approximate Pattern Matching

Given a text T, a pattern P, and a threshold k, identify the (starting positions of) substrings of T that are at **edit distance** of at most k to P.

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Given a text T, a pattern P, and a threshold k, identify the (starting positions of) substrings of T that are at **edit distance** of at most k to P.

Edit distance: minimum number of insertions, deletions, or substitutions of single characters to transform one string into another string

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Given a text T, a pattern P, and a threshold k, identify the (starting positions of) substrings of T that are at **edit distance** of at most k to P.

Applications

- **Text Retrieval**: Find occurrences of patterns or phrases in a text, accounting for misspellings or conversion errors
- **Signal Processing**: Find patterns in signals, accounting for transmission errors
- **Computational Biology**: Find specific patterns in DNA sequences, accounting for mutations or evolutionary alterations

early 1980's

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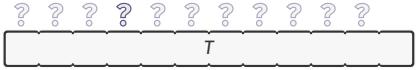

Focus: Obtain starting positions of occurrences

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early 1980's

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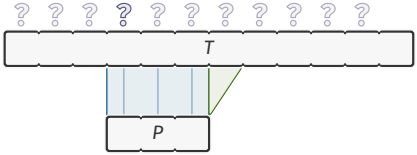

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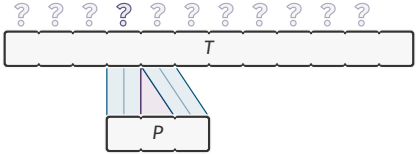

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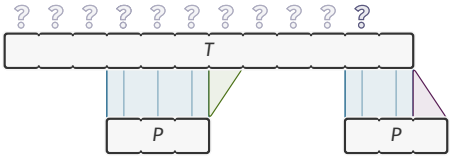

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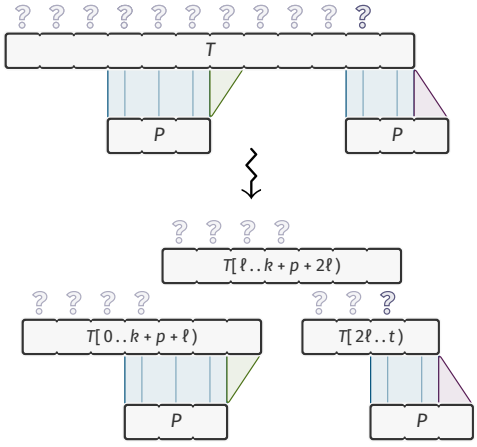
early 1980's

Given: text T, pattern P, threshold k; Find: (starting pos. of) substr. of T at edit distance $\leq k$ to P.

Focus: Obtain starting positions of occurrences

"Standard Trick": write $t := |T|$, $p := |P|$
Split T into overlapping fragments of len $\ell + p + k$

$\rightsquigarrow O(t/\ell)$ instances,
each "responsible" for its first ℓ positions



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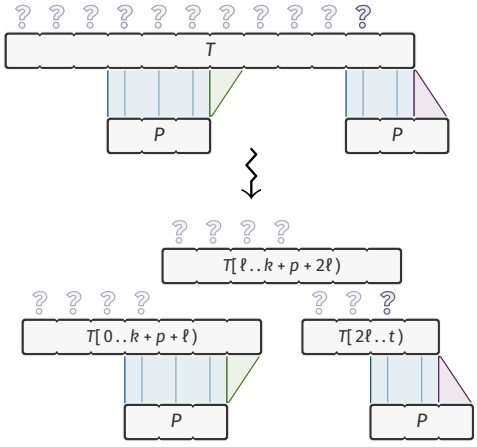
$\rightsquigarrow$ Useful special cases: $\ell = k$ and $\ell = 0.5\,p$



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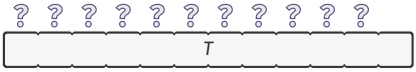

"Filter and Verify Paradigm"

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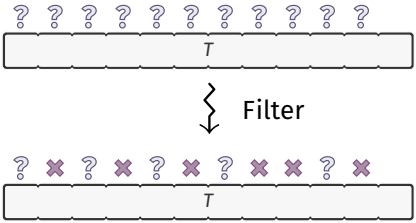
early 1980's

Given: text T, pattern P, threshold k; Find: (starting pos. of) substr. of T at edit distance $\leq k$ to P.

"Filter and Verify Paradigm"

Step 1, Filter: (typically fast)
Compute (small) superset of starting positions



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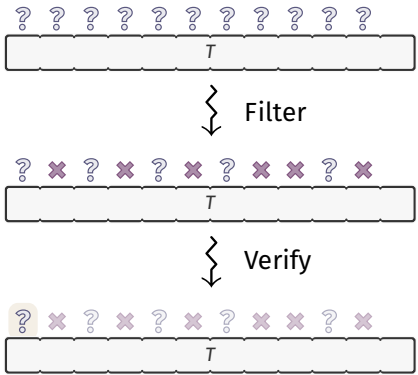
Check for occ at each remaining position



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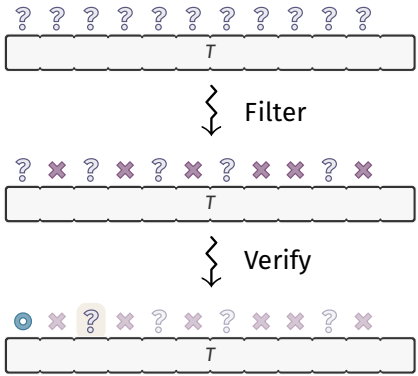

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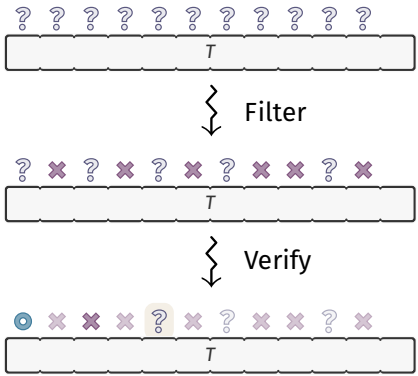

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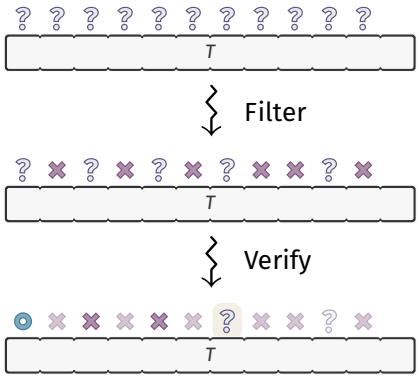

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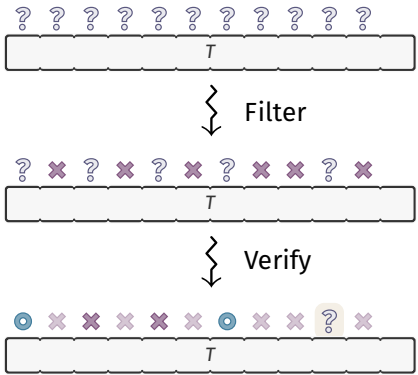

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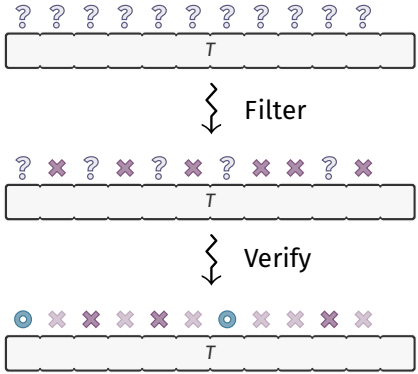

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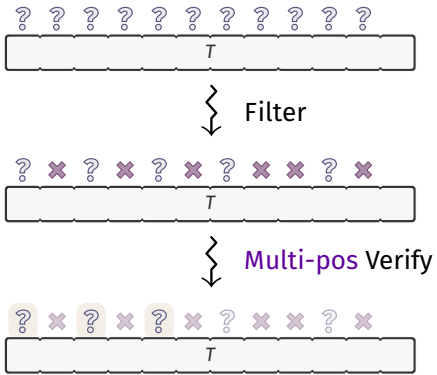
Step 2', Multi-pos Verify: (typically slow)
Check for occ at each remaining position, multiple positions at once



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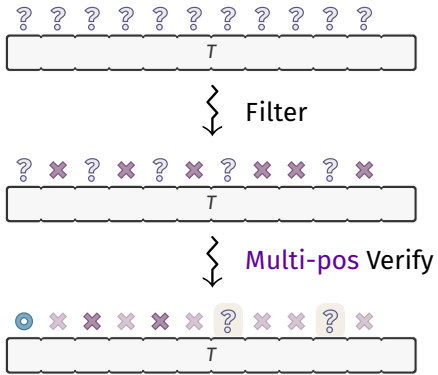

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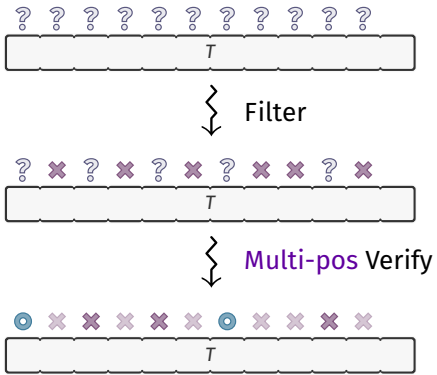

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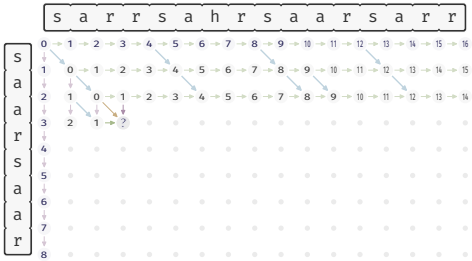
Edit dist/Verify Algorithm	APM Algorithm
Textbook DP $\rightsquigarrow O(tp)$	Verify pos 1 by 1 $\rightsquigarrow O(t^2p)$
[Sellers, 1980] and others	

$t := |T|, p := |P|$



$e_{i,0} = i$ (delete $T[0..i)$), $e_{0,j} = j$ (insert $P[0..j)$)

$$e_{i,j} = \min \begin{cases} e_{i-1,j} + 1 & \text{(del from } T\text{)} \\ e_{i,j-1} + 1 & \text{(ins in } T\text{)} \\ e_{i-1,j-1} + [T[i] \neq P[j]] & \text{(match/subst)} \end{cases}$$

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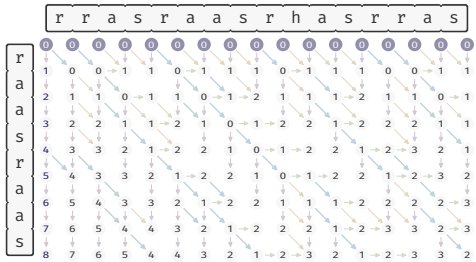

$e_{i,0} = 0$ (delete $T[0..i)$), $e_{0,j} = j$ (insert $P[0..j)$)

$$e_{i,j} = \min \begin{cases} e_{i-1,j} + 1 & \text{(del from } T\text{)} \\ e_{i,j-1} + 1 & \text{(ins in } T\text{)} \\ e_{i-1,j-1} + [T[i] \neq P[j]] & \text{(match/subst)} \end{cases}$$

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Compute DP diagonals, jump over equal substr in $O(1)$ time $\rightsquigarrow O(t + k^2)$ [Landau, Vishkin'89]	Verify pos 1 by 1 $\rightsquigarrow O(tk^2)$ Verify $\ell \leq t$ pos at once $\rightsquigarrow O(t/\ell \cdot (\ell + k)k) = O(tk)$

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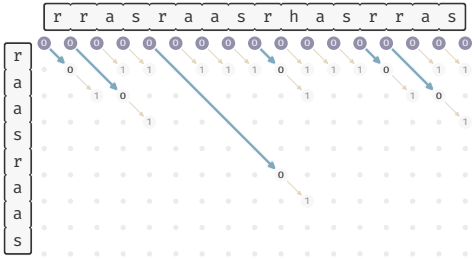

Compute furthest pos on diag d reachable w/ $0, \ldots, k$ edits.
$\rightsquigarrow$ Jump over equal substrings in $O(1)$ time

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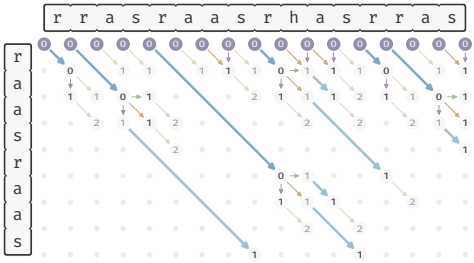

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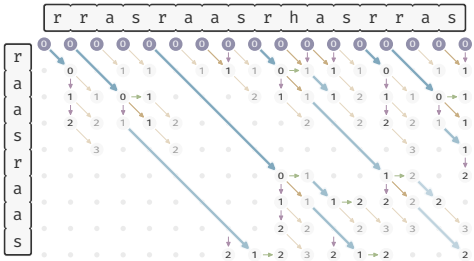

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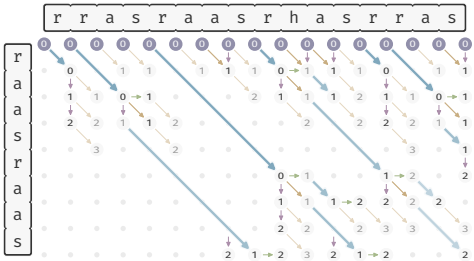
$t := |T|, p := |P|$

$\rightsquigarrow$ Didn't use filter...



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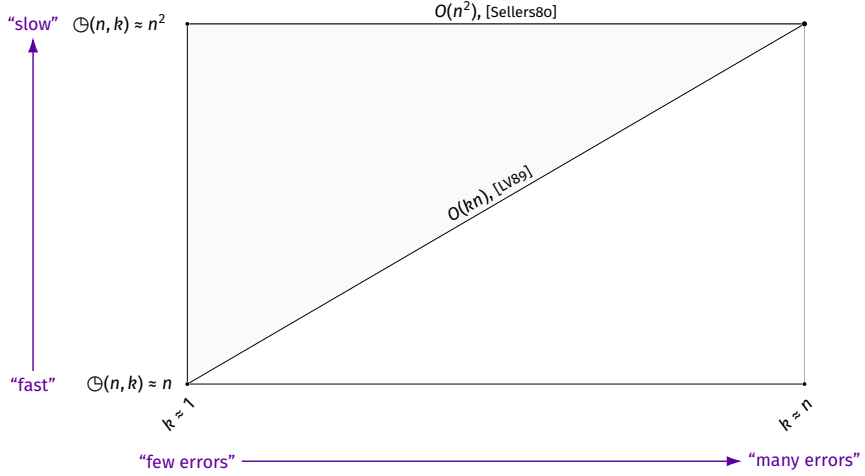
State-of-the-Art Algorithms



Existing algorithms for the case $n \approx t \approx 2p$

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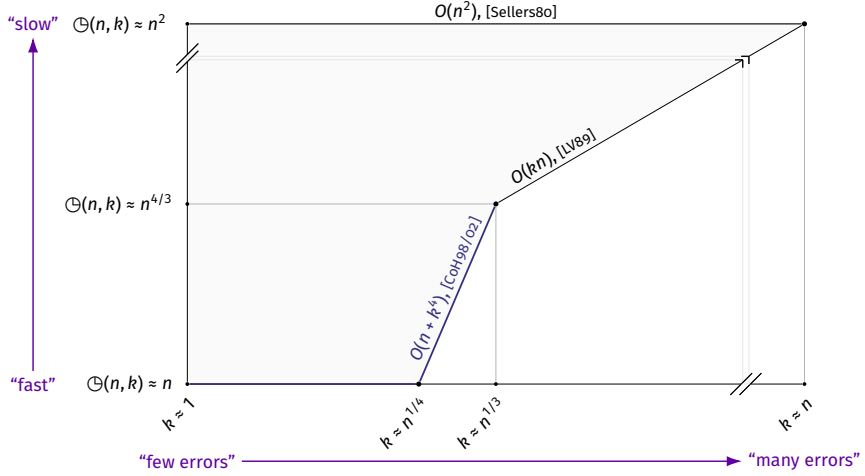

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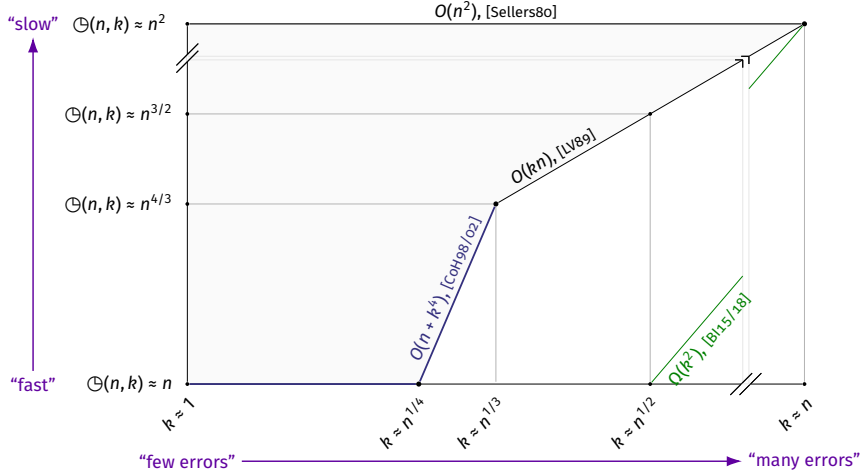

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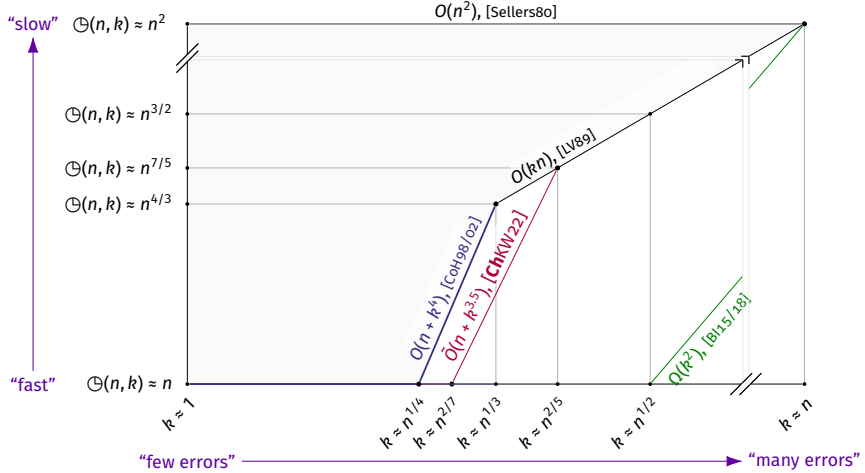
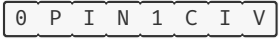


Existing algorithms for the case $n \approx t \approx 2p$

How similar are two strings X and Y?



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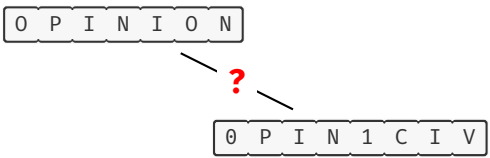

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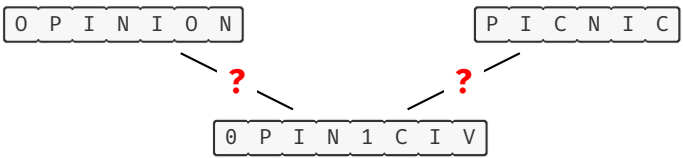

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Min number of character insertions, deletions, and substitutions that transform X to Y

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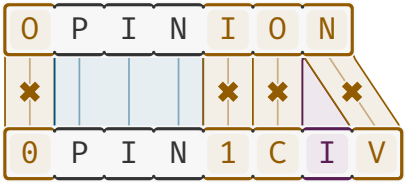

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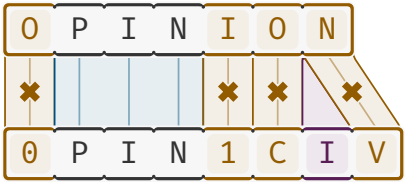

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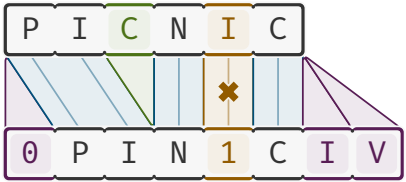

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$\mathrm{ED}^w(X, Y)$

Weighted Edit Distance

Min cost of transforming X to Y using character edits, where:

- inserting y costs $w(\varepsilon, y)$;
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$w(0, O) := 1 \quad w(1, I) := 1 \quad w(C, O) := 1 \qquad w(*, *) := 2 \quad w(*, \varepsilon) := 1 \quad w(\varepsilon, *) := 10$

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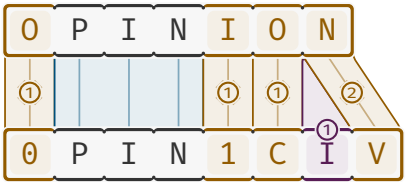

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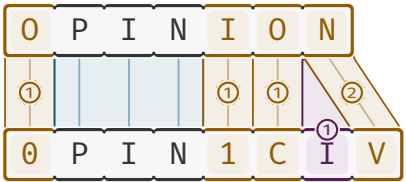

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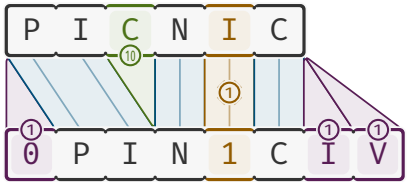

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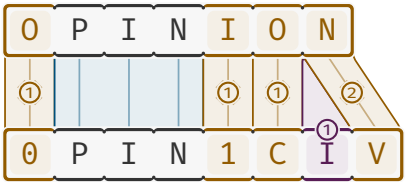

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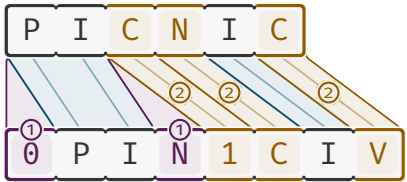

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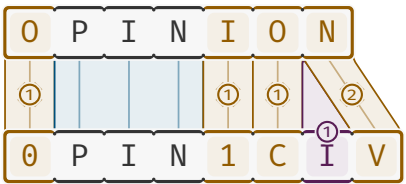

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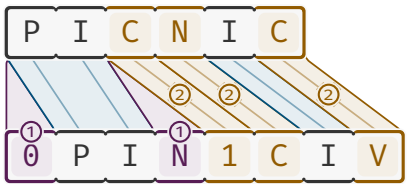

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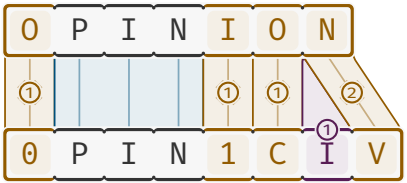

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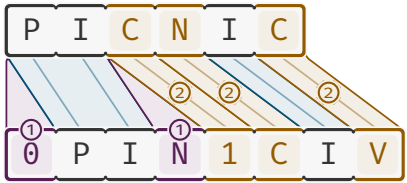

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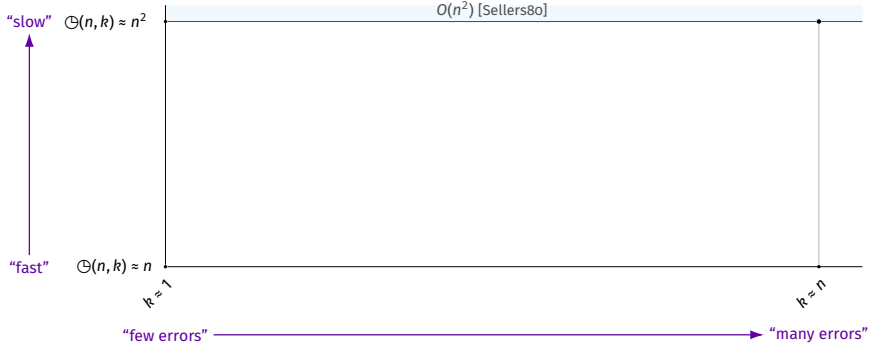
Otherwise: could scale weights and parameterizing the running time by k does not make sense.

Weighted Approximate Pattern Matching

Given: T, P, k, oracle to w; Find: (starting pos. of) substr. of T at weighted ED $\leq k$ to P.



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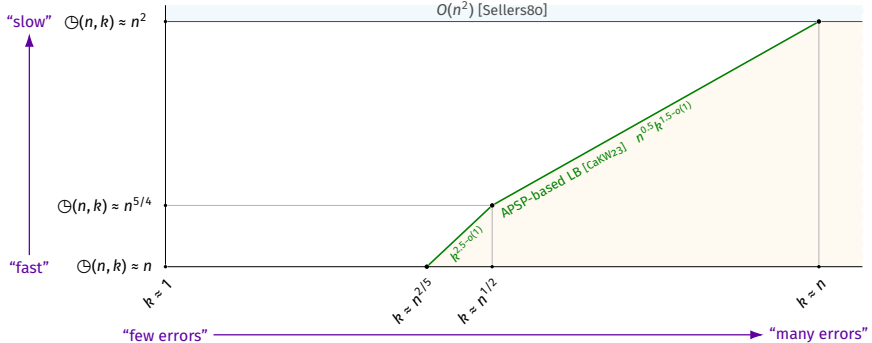

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Some (unsatisfactory) upper bounds of the forms $t \cdot k^{O(1)}$ and $t \cdot \sqrt{p} \cdot k^{O(1)}$ follow from the state-of-the-art algorithm for weighted edit distance computation [Cassis, Kociumaka, Wellnitz'23].

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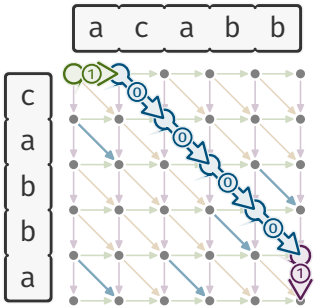
We nearly-match the best known upper bounds for the unweighted setting for a large range of parameters.

Main Challenge: Can't be Greedy



In the unweighted setting, when we see a long match, we greedily take it. This is the basis of the $O(kt)$-time algorithm we saw before.

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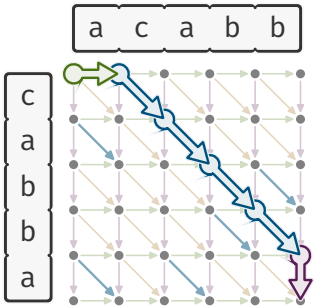

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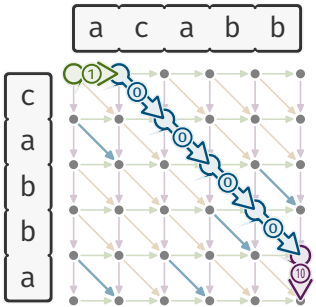

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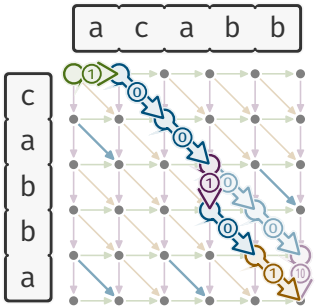

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What about in the weighted setting? A disaster.

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Observation: $\mathrm{ED}^w(X, Y) \geq \mathrm{ED}(X, Y)$ (by norm.)

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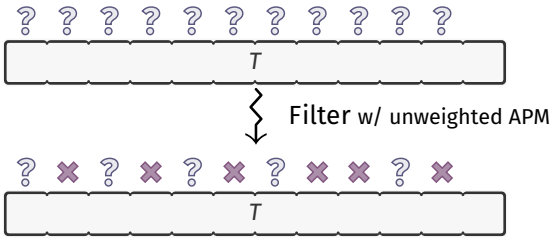

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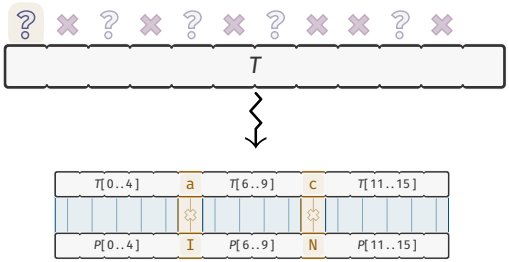

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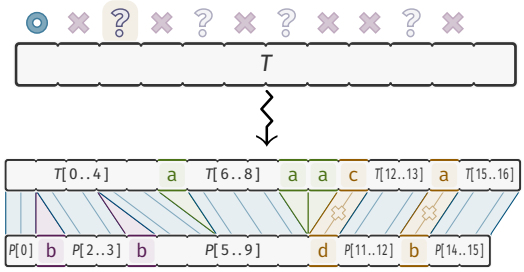

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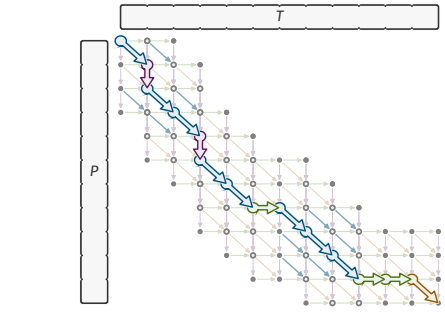

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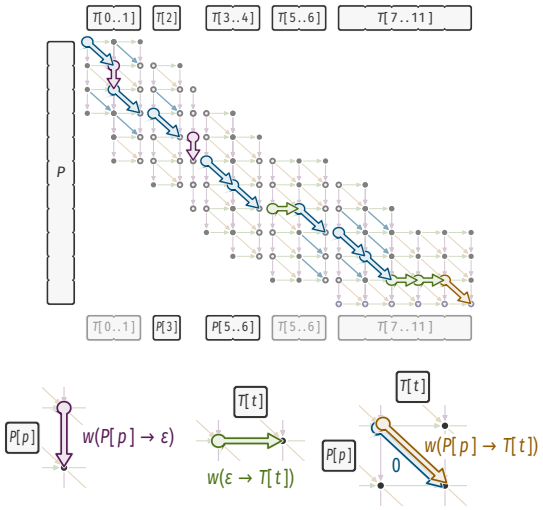

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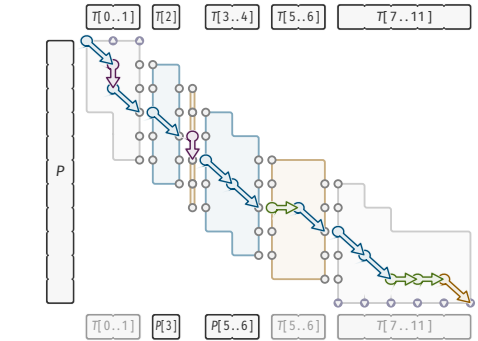

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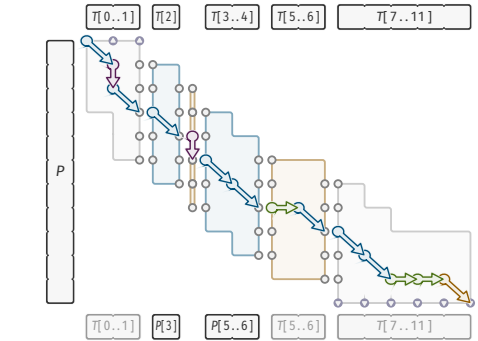
$\rightsquigarrow$ $\tilde{O}(kt)$ total



Summary of our Results

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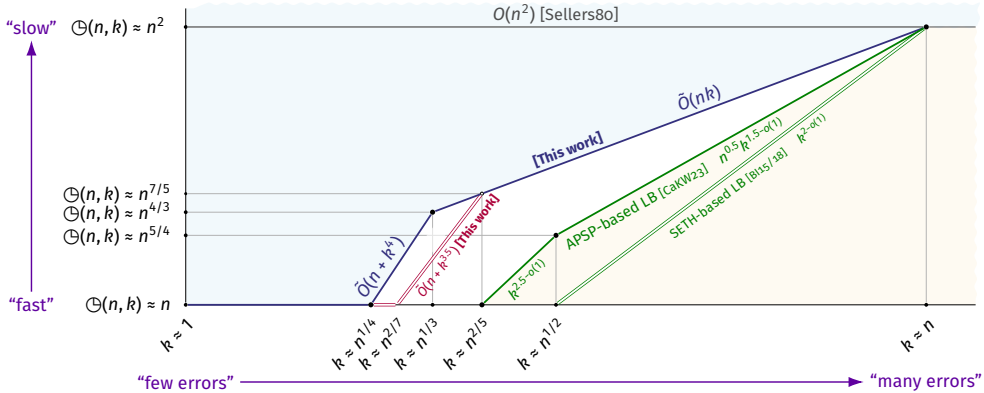
We can do better when the weights are small integers, matching the state of the art for the unweighted setting.

- Close gaps between UBs and LBs; there is a combinatorial barrier in improving the $\tilde{O}(t + k^4 \cdot t/p)$-time algorithm
- Report substrings instead of starting positions; we can do $\tilde{O}(k^2 t)$



Thank you!

Full version: https://arxiv.org/abs/2510.17752